Influence of charge variation on particle oscillations in the plasma sheath

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The theory of dust particle oscillations in the plasma sheath is presented, taking into account particle charging kinetics and neutral gas friction. Effects of ''regular'' and stochastic charge variations are considered. It is shown that whilst regular variations generally enhance the damping of horizontally propagating dust lattice waves, they can also cause an instability in the vertical oscillations of single particles. The stochastic charge variations, if sufficiently strong, result in exponential growth of the mean energy of both types of oscillations.

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I. INTRODUCTION

Oscillations of charged dust particles trapped in a low temperature plasma sheath have been considered in detail in many publications [1–14]. The most interesting types of oscillations are the vertical oscillations of single particles [1–3] and the horizontal dust lattice (DL) waves [4,5]. Experimental study of these oscillations allows us to evaluate basic parameters of the system—the particle charge, interparticle coupling parameter, etc. [8–10]. It is usually assumed that particles have an equilibrium charge, which is determined by the balance of the average ion and electron fluxes on the particle surface. However, the actual charge varies randomly around this mean value, because the fluxes are discrete. The particle charge distribution is then a stationary Gaussian, and its dispersion is directly proportional to the mean particle charge [15,16]. In addition, the electron temperature and plasma potential (density) fluctuate locally in any real plasma. Since the particle size is much less than the electron Debye length, this ''noise'' can significantly increase the magnitude of the random charge fluctuations [17].

Along with the stochastic charge fluctuations, there are also ''regular'' variations caused by the particle motion. In the plasma sheath, the equilibrium particle charge is a strong function of distance from the electrode surface, and thus vertical oscillations are always accompanied by regular charge variations. In sufficiently dense plasma crystals, the horizontal DL waves also produce regular charge variations. There are two independent reasons for that. The first one is the charge variation due to the mutual decrease of electron fluxes as particles approach each other [18]. This mechanism could be especially important in a two-dimensional (2D) layer as a result of surface density perturbations. The second mechanism arises in 3D crystals: It is caused by a depression of the electron-to-ion density ratio due to the local decrease of plasma potential in regions of increased dust (volume) density [19]. Below we focus on the case of a 2D plasma crystal, and thus on the first mechanism of regular charge variation.

The important peculiarity of the charging process is that the particle charging time is finite (''delayed'' charging). This implies that the charge never reaches its equilibrium value (corresponding to the instantaneous position of the particle), even if we consider only regular variations. Recent experiments by Nunomura *et al.* [12] performed in a low-pressure dc discharge show that the delayed particle charging can result in an instability of the vertical oscillations. In the absence of the neutral gas friction this type of instability was predicted by Nitter *et al.* [1]. In addition, Morfill *et al.* [11] demonstrated theoretically that the stochastic charge variations may cause an instability of the DL wave for sufficiently small gas pressure. Numerical simulations of particle heating by the charge fluctuations were performed by Vaulina *et al.* [14], but the stochastic modulation of the vertical resonance frequency was not taken into account. In this paper, we consider both vertical oscillations of single particles and horizontal DL waves, and we study separately the role of regular and stochastic charge variations, taking into account the charging kinetics and gas friction. We assume a pressure to be sufficiently small, so that the particle oscillations are weakly damped (otherwise, the charge variation effects are ''hidden'' by strong damping due to the friction).

II. REGULAR CHARGE VARIATION

In this section we neglect the stochastic charge fluctuations and consider only regular variations caused either by relative horizontal displacements, or by vertical motion.

Horizontal oscillations (DL wave): In order to describe the DL wave of small amplitude we apply the simplified model of the 1D particle string [4,5,11]. In a steady state, particles of the mass M and charge $Q_0<0$ are separated by the distance Δ and interact via a shielded Coulomb potential; the screening length approximately equals the electron Debye length λ_{De}. For simplicity, we assume $\Delta \gtrsim \lambda_{De}$, i.e., only the nearest neighbor interactions are essential. Perturbations of particle position in the wave result in a variation of the charges. Let us present the charge of each particle as $Q_n = Q_0 + \delta Q_n$, where $\delta Q_n \sim O(y_n, y_{n\pm1})$, and y_n is the dimensionless displacement of the nth particle. Then, we can directly use the equations for the dimensionless displacement derived in Ref. [11] [Eqs. (2) and (3)]. Neglecting terms $O(y_n^2, y_n y_{n\pm1})$, we obtain

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$$\ddot{y}_n + 2\gamma\dot{y}_n + \Omega_{\rm DL}^2(y_n - y_{n+1} - y_{n-1}) = \frac{\eta}{1+\eta^2}\Omega_{\rm DL}^2(\delta\tilde{Q}_{n-1} - \delta\tilde{Q}_{n+1}), \qquad (1)$$

where γ is the damping rate due to neutral gas friction [20], $\eta = 1 + \Delta/\lambda_{De}$, the displacement y_n is normalized to Δ, $\delta\tilde{Q}_{n\pm1} = \delta Q_{n\pm1}/Q_0$ is the dimensionless charge variation, and the DL frequency scale is

$$\Omega_{\rm DL}^2 = \frac{Q_0^2}{M\Delta^3}(1+\eta^2)e^{1-\eta}.$$

The kinetic equation for the particle charge is of the form

$$\dot{Q}_n = -\Omega_{\rm ch}(Q_n - Q_n^{\rm (eq)}), \qquad (2)$$

where $\Omega_{\rm ch}$ is the steady-state charging frequency of a particle [21]. The ''instantaneous'' equilibrium charge, $Q_n^{\rm (eq)}$, is a function of the displacement of the nth particle with respect to the $(n+1)$th and $(n-1)$th particles. Assuming $|y_n - y_{n\pm1}| \ll 1$, we can approximate these dependencies by the factors $1+\alpha(y_{n+1}-y_n)$ and $1+\alpha(y_n - y_{n-1})$, respectively, or

$$Q_n^{\rm (eq)} \simeq Q_0[1+\alpha(y_{n+1}-y_{n-1})], \qquad (3)$$

where $\alpha > 0$ is some coefficient [18]. Substituting Eq. (3) in Eq. (2), we obtain the following equation for the charge variation

$$\delta\dot{Q}_n = -\Omega_{\rm ch}[\delta Q_n - \alpha Q_0(y_{n+1}-y_{n-1})]. \qquad (4)$$

For the traveling wave, all the variables are proportional to $\exp\{i(\omega t + Kn\Delta)\}$, where K is the wave vector, $-\pi \leqslant K\Delta \leqslant \pi$. Substituting this form for y_n and $\delta\tilde{Q}_n$ in Eqs. (1) and (4), we get the dispersion relation for the DL wave with the regular charge variation and delayed charging,

$$(i\omega + \Omega_{\rm ch})[-\omega^2 + 2i\gamma\omega + 2\Omega_{\rm DL}^2(1-\cos K\Delta)] = 2\mathcal{A}\Omega_{\rm DL}^2\Omega_{\rm ch}(1-\cos 2K\Delta), \qquad (5)$$

where $\mathcal{A} = \alpha\eta/(1+\eta^2)$ is a measure of regular charge variations in the wave. The derived dispersion relation has simple structure. The left-hand side is the product of two factors. The first one, $(i\omega + \Omega_{\rm ch})$, represents the dust charging (DCh) branch [22] and describes the charge variation decay, the second one represents the DL branch. The right-hand side of Eq. (5) describes the coupling of these branches. We assume that the damping of the DL oscillations due to neutral friction is small, $\mathrm{Re}\,\omega \gg \gamma$, and that the dust charging branch is weakly coupled with the DL branch, i.e., $\Omega_{\rm ch} \gg |\omega|$ [22]. Then $\mathrm{Re}\,\omega \gg \mathrm{Im}\,\omega$, and Eq. (5) can be solved approximately for the DL branch. Extracting the real and imaginary parts in Eq. (5) we obtain the solution for the frequency $\mathrm{Re}\,\omega$ and damping rate $\mathrm{Im}\,\omega$ of the DL wave,

$$(\mathrm{Re}\,\omega)^2 \simeq 2\Omega_{\rm DL}^2[(1-\cos K\Delta) - \mathcal{A}(1-\cos 2K\Delta)] \equiv \omega_0^2,$$

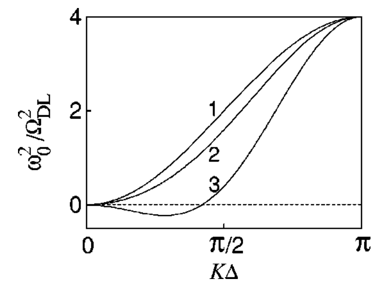

FIG. 1. Squared frequency of the DL wave with regular charge variations, $\omega_0^2/\Omega_{\rm DL}^2$ [normalized to the DL frequency scale $\Omega_{\rm DL}$, see Eq. (6)] vs the dimensionless wave vector, $K\Delta$. The curves correspond to $\mathcal{A}=0$ (1), $\mathcal{A}=0.1$ (2), and $\mathcal{A}=0.4$ (3).

$$\mathrm{Im}\,\omega \simeq \gamma + \mathcal{A}\frac{\Omega_{\rm DL}^2}{\Omega_{\rm ch}}(1-\cos 2K\Delta). \qquad (6)$$

Figure 1 shows the dependence of $\omega_0^2/\Omega_{\rm DL}^2$ on $K\Delta$ for different values of the parameter $\mathcal{A}$. This parameter grows rapidly with the dust density. While the density is small and $\mathcal{A} < 1/4$, the phase velocity of long waves decreases with $\mathcal{A}$ as $C_{\rm DL}|_{K\to0} \simeq \sqrt{1-4\mathcal{A}}\Delta\Omega_{\rm DL}$. For sufficiently high density, when $\mathcal{A} > 1/4$, long-wavelength perturbations are unstable and grow exponentially without oscillations $[(\mathrm{Re}\omega)^2 < 0]$. This is because the charge decreases too rapidly and the energy of interparticle coupling diminishes as particles approach. The delayed charging ($\Omega_{\rm ch}^{-1}$ is finite) does not affect the frequency of oscillations, but increases the damping rate in Eq. (6) (see Fig. 2). Note, that the damping rate is maximal at $K\Delta = \pi/2$ and it tends to γ for $K\Delta \to 0,\pi$. The physical background of the delayed charging damping is considered in the next subsection.

Vertical oscillations: The equation of vertical motion for a single particle is

$$\ddot{z} + 2\gamma\dot{z} = \frac{QE}{M} - g, \qquad (7)$$

where E is the electric field, $E<0$. The particle oscillates around the steady-state position $z=0$, $E(0)=E_0$ and $Q^{\rm (eq)}(0) = Q_0 < 0$. For sufficiently small amplitude of oscillations we can set $E \simeq E_0 + E_0' z$ (a prime denotes the derivative at $z=0$). Substituting this expression together with $Q = Q_0 + \delta Q$ in Eq. (7) and using the condition $Q_0E_0 = Mg$, we obtain

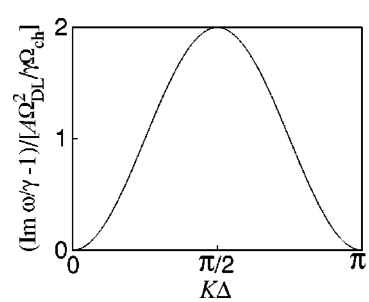

FIG. 2. Relative variation of the damping rate of the DL wave with regular charge variations, $(\mathrm{Im}\,\omega/\gamma - 1)$ [normalized to the friction damping γ, see Eq. (6)], vs the dimensionless wave vector, $K\Delta$, in units of $\mathcal{A}\Omega_{\rm DL}^2/\gamma\Omega_{\rm ch}$.

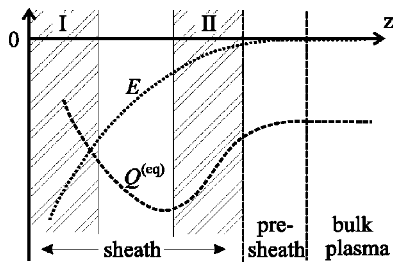

FIG. 3. Qualitative dependence of the equilibrium particle charge, $Q^{(\mathrm{eq})}$, and the vertical electric field in a plasma, E, as function of the distance z from the lower electrode. In regions I and II the particle motion can be unstable.

$$\ddot{z}+2\gamma\dot{z}-\frac{Q_0E_0'}{M}z=g\,\delta\tilde{Q}. \tag{8}$$

The kinetic equation for the charge is similar to Eq. (2), $\dot{Q}=-\Omega_{\mathrm{ch}}(Q-Q^{(\mathrm{eq})})$, where the equilibrium charge is

$$Q^{(\mathrm{eq})}\simeq Q_0+Q_0'z. \tag{9}$$

Thus, we get the following kinetic equation for the charge variation:

$$\delta\dot{Q}=-\Omega_{\mathrm{ch}}(\delta Q-Q_0'z). \tag{10}$$

For a harmonic solution of Eqs. (8) and (10) we obtain

$$(i\omega+\Omega_{\mathrm{ch}})\left(-\omega^2+2i\gamma\omega-\frac{Q_0E_0'}{M}\right)=\frac{Q_0'E_0}{M}\Omega_{\mathrm{ch}}. \tag{11}$$

The structure of Eq. (11) is completely similar to that of the dispersion relation (5): The left-hand side represents the DCh branch and the branch of vertical oscillations, and the right-hand side describes the coupling between them. Solving the obtained equation with the same assumptions, we get approximate expressions for the frequency and damping rate of the oscillations,

$$(\mathrm{Re}\,\omega)^2\simeq-\frac{(QE)_0'}{M}\equiv\Omega_{\mathrm{v}}^2,$$

$$\mathrm{Im}\,\omega\simeq\gamma-\frac{1}{2}\left(\frac{Q_0'E_0}{(QE)_0'}\right)\frac{\Omega_{\mathrm{v}}^2}{\Omega_{\mathrm{ch}}}, \tag{12}$$

where Ω_{v} is the eigenfrequency of the vertical oscillations. Figure 3 shows a qualitative dependence of $Q^{(\mathrm{eq})}$ and E as functions of the vertical position z. The charge is practically independent of z in the bulk plasma, but as the electrode is approached it decreases rapidly ($|Q^{(\mathrm{eq})}|$ increases) in the presheath and just below the sheath edge. At even smaller z, the charge attains a minimum and then starts increasing. Normally, particles are trapped near the sheath edge, where $(QE)_0'<0$. But, if the amplitude of oscillations is too large and a particle enters the region I, where $(QE)_0'>0$, the motion becomes unstable $[(\mathrm{Re}\,\omega)^2<0]$ and the particle drops onto the electrode.

There is one more region of instability, where $-Q_0'E_0$ is positive and exceeds a certain threshold, so that the damping rate $\mathrm{Im}\,\omega$ is negative (region II). This instability was observed and explained qualitatively by Nunomura *et al.* [12]. Due to delayed charging, the particle motion is not a potential one (even in the absence of friction). On the way down, $|Q(z)|$ is always less than the equilibrium value $|Q^{(\mathrm{eq})}(z)|$, whereas on the way up the opposite inequality holds. Therefore, the particle gains energy during the whole circle of oscillation, and if this exceeds the energy dissipation due to friction, then oscillations are unstable. Note that in the absence of friction the condition of the instability $Q_0'E_0<0$ was obtained for the first time by Nitter *et al.* [1].

In contrast with the vertical oscillations, the (horizontal) DL wave is damped due to delayed charging [see Eq. (6)]. The physics of this damping is the same as that of the instability considered above. As particles approach in the DL wave, the absolute value of their equilibrium charge (3) decrease. Therefore, $|Q_n|>|Q_n^{(\mathrm{eq})}|$ during compression, and the opposite during rarefaction, so that delayed charging causes particles to lose energy in the DL oscillations.

III. STOCHASTIC CHARGE VARIATION

Let us consider the role of stochastic fluctuations. We assume that the random variations of charge are much stronger than the regular ones, so that the kinetic of stochastic variations only is taken onto account. This approach allows us to determine the conditions when one or the other kind of the variation is more important. The charge is no longer a self-consistent variable, but is an independent random function with certain stochastic properties. Below we assume that the random charge variation is a stationary process, and the mean charge equals the equilibrium value.

Horizontal oscillations (DL wave): We can present the charge of each particle as $Q_n=\langle Q_n\rangle+\delta Q_n(t)$, where the average charge $\langle Q_n\rangle$ equals $Q_n^{(\mathrm{eq})}$ from Eq. (3) and $\delta Q_n(t)$ is a random function. Let us consider a solution of the form $y_n=y(t)\exp(iKn\Delta)$. Using results of Ref. [11] [Eqs. (2) and (3)] we readily obtain the following stochastic equation for the amplitude of the traveling DL wave with a randomly varying particle charge:

$$\ddot{y}+2\gamma\dot{y}+\omega_0^2[1+\xi(t)]y=f(t), \tag{13}$$

where ω_0 is determined by Eq. (6) and the random functions are

$$\xi(t)=\delta\tilde{Q}-\left(\frac{\Omega_{\mathrm{DL}}}{\omega_0}\right)^2[\delta\tilde{Q}_+(\mathrm{e}^{iK\Delta}-1)+\delta\tilde{Q}_-(\mathrm{e}^{-iK\Delta}-1)],$$

$$f(t)=\frac{\eta}{1+\eta^2}\Omega_{\mathrm{DL}}^2(\delta\tilde{Q}_--\delta\tilde{Q}_+). \tag{14}$$

Here $\delta\tilde{Q}\equiv\delta\tilde{Q}_n$, $\delta\tilde{Q}_+$, and $\delta\tilde{Q}_-$ denote the dimensionless charge fluctuations on the ''central,'' ''right,'' and ''left'' particles, respectively. The only difference of Eq. (13) from the similar equation derived in Ref. [11] is that now we take into account the regular variations (assuming them to be equilibrium), which change the wave frequency ω_0 and the random function $\xi(t)$.

We suppose that correlations of the charge fluctuations on neighboring particles are negligible, $\langle \delta\tilde{Q}(t)\delta\tilde{Q}_{\pm}(t-\tau)\rangle \simeq 0$. This assumption is reasonable, since the spatial scale of plasma fluctuations (which determine the charge fluctuations) is always less than the electron Debye length λ_{De}, whereas the interparticle distance $\Delta \gtrsim \lambda_{De}$. In the Langevin approach for particle charging, the autocorrelation function is [15]

$$\langle \delta\tilde{Q}(t)\delta\tilde{Q}(t-\tau)\rangle \simeq \tilde{\sigma}^2 \exp(-\Omega_{\rm ch}\tau), \tag{15}$$

where $\tilde{\sigma}^2$ is the dimensionless dispersion of the charge distribution (normalized to Q_0^2). Note that the autocorrelation time is the inverse charging frequency. This result is natural, since relaxation of both regular and stochastic charge perturbations is described by the same kinetic equation.

Of course, we cannot solve Eq. (13) exactly, since it is a stochastic differential equation, and each solution of this equation is determined by a particular realization of the random variable ξ. However, properties of the stochastic process $y(t;[\xi])$ can be studied using the approximate method of expansion over small Kubo number [11]. This method allows us to derive nonstochastic equations for moments of y: mean displacement, $\langle y\rangle$, mean squared displacement and velocity, $\langle yy^*\rangle$ and $\langle \dot{y}\dot{y}^*\rangle$, etc. In principle, the first two moments of y—the average and dispersion—allow us evaluate the main peculiarities of the stochastic process. We can directly use the results of Ref. [11], replacing ω_0 and $\xi(t)$ with those from Eqs. (13) and (14). It was shown in Ref. [11] that for reasonable parameters of a discharge, the equation for $\langle y\rangle$ is that of a damped harmonic oscillator with the damping rate $\simeq\gamma$ and frequency $\simeq\omega_0$. However, equations for the second moments can be unstable [11]. Actually, functions $\langle yy^*\rangle$ and $\langle \dot{y}\dot{y}^*\rangle$ determine the mean energy $\langle \mathcal{E}\rangle$ of the DL wave. It implies, that the plasma crystal can be unstable energy wise, due to the random charge variations. The mean energy changes with time as $\langle \mathcal{E}(t)\rangle \propto \exp(-\Gamma_{\mathcal{E}}t)$, with the ''energy damping rate''

$$\frac{1}{2}\Gamma_{\mathcal{E}} \simeq \gamma - \frac{1}{2}\tilde{\sigma}^2\frac{\Omega_{\rm DL}^2}{\Omega_{\rm ch}}\left[\left(\frac{\omega_0}{\Omega_{\rm DL}}\right)^2 + 4\left(\frac{\Omega_{\rm DL}}{\omega_0}\right)^2(1-\cos K\Delta)\right]. \tag{16}$$

Using expression for $\omega_0^2/\Omega_{\rm DL}^2$ from Eq. (6), we plot the relative variation of the damping rate, $\frac{1}{2}\Gamma_{\mathcal{E}}/\gamma - 1$, versus $K\Delta$ for different values of the parameter $\mathcal{A}$ (see Fig. 4). In contrast with regular charge variations [which increase the DL damping, see Eq. (6) and Fig. 2], the random variations can induce the instability at low pressures, and their influence does not vanish for $K\Delta \to 0,\pi$.

Vertical oscillations: The vertical particle oscillations with a randomly varying charge are described by Eq. (7) with the charge $Q = \langle Q\rangle + \delta Q(t)$, where $\langle Q\rangle$ equals $Q^{(\rm eq)}$ from Eq. (9). Using a linear expansion of the electric field, we get the stochastic equation

$$\ddot{z} + 2\gamma z + \Omega_{\rm v}^2\left(1 + \frac{Q_0E_0'}{(QE)_0'}\delta\tilde{Q}(t)\right)z = g\,\delta\tilde{Q}(t). \tag{17}$$

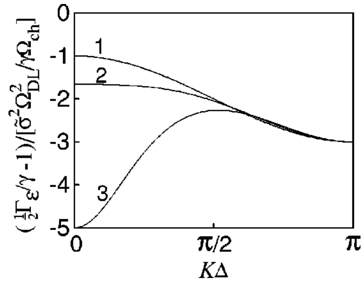

FIG. 4. Relative variation of the ''energy damping rate'' of the DL wave with stochastic charge fluctuations, $(\frac{1}{2}\Gamma_{\mathcal{E}}/\gamma - 1)$ [normalized to the friction damping γ, see Eq. (16)] vs the dimensionless wave vector, $K\Delta$, in units of $\tilde{\sigma}^2\Omega_{\rm DL}^2/\gamma\Omega_{\rm ch}$. The curves correspond to $\mathcal{A}=0$ (1), $\mathcal{A}=0.1$ (2), and $\mathcal{A}=0.2$ (3).

Equation (17) has real coefficients and therefore is simpler for analysis than Eq. (13). Using results from Ref. [23], we readily obtain that the mean amplitude $\langle z\rangle$ obeys the equation of a damped harmonic oscillator, as well. The corresponding damping rate is $\simeq\gamma$ and the frequency $\simeq\Omega_{\rm v}$. The equation for the second moments gives the following threshold of the energy-wise instability:

$$\frac{1}{2}\Gamma_{\mathcal{E}} \simeq \gamma - \frac{1}{2}\left(\tilde{\sigma}\frac{Q_0E_0'}{(QE)_0'}\right)^2\frac{\Omega_{\rm v}^2}{\Omega_{\rm ch}}. \tag{18}$$

The ''energy damping rate'' (18) has the same structure as that in Eq. (16).

IV. DISCUSSION AND CONCLUSIONS

Although both the regular and stochastic variations of charge have the same relaxation time, $\Omega_{\rm ch}^{-1}$, the effect of these variations is different. If the damping rate due to friction is sufficiently small, random fluctuations of charge result in an instability of both types of oscillations—vertical and horizontal. In contrast, the regular variations can cause an instability of the vertical oscillations, but increase the damping rate of the DL wave.

The reason for this difference is the following: The regular charge variation is a self-consistent variable depending on the particle displacement; the sign of the work done by the nonpotential force due to delayed charging (averaged over a period of oscillation) is determined by the signs of the coefficients Q_0' and α in the expansion of the equilibrium charge [see Eqs. (3) and (9)]. For vertical oscillations, the work $\propto Q_0'$ is positive, for the DL wave the work is proportional to $-\alpha$ and is negative. In the case of stochastic charge variations, the particle motion is governed by equations of a parametric oscillator [see Eqs. (13) and (17)] with a randomly varying frequency. For both types of oscillations (vertical and horizontal), the time dependence of the frequency is characterized by the stochastic variation of charge, $\delta\tilde{Q}(t)$. The sign of the average work of the random force $\delta\tilde{Q}(t)\mathbf{r}$ (where $\mathbf{r}$ is y or z) equals the sign of the spectral density $\mathcal{S}(\omega)$ of the charge variation at $\omega = 2\omega_0$ [24]. For any stationary random process with a monotonic autocorrelation function [in particular, with that from Eq. (15)], $\mathcal{S}(\omega) > 0$ at any ω, so that the random fluctuations of charge always cause the mean energy of oscillations to grow.

Comparing expressions for damping rates obtained for the regular and stochastic fluctuations, we can judge which kind of the charge variation might cause an instability (or damping) of oscillations in the case of a finite $\Omega_{\rm ch}^{-1}$. Let us consider typical experimental conditions of discharges: argon plasma at pressures $p\sim 1$–10 Pa, plasma number density $n_{e,i}\sim 10^7$–10^9 cm^{-3}, electron temperature $T_e\sim 1$ eV, electron Debye length $\lambda_{\rm De}\sim 10^2$–$10^3$ μm, particle radius $a\sim 1$–10 μm, particle charge number $Z\sim 10^3$–10^4. The value of the charging frequency in the region of the sheath edge has been estimated as $\Omega_{\rm ch}\sim a c_s/\lambda_{\rm De}^2$ [11], where $c_s=\sqrt{T_e/m_i}$ is the ion acoustic velocity. For our conditions, $\Omega_{\rm ch}\sim 10^3$–10^6 s^{-1}. The friction damping is $\gamma\sim 0.3$–3 s^{-1} [20].

First, we investigate the DL wave. From Eqs. (6) and (16) for damping rates we see that the regular (random) variations of charge are more important when the scale of regular variations, $\mathcal{A}$, is much greater (smaller) than that of random variations, $\tilde{\sigma}^2$. Unfortunately, processes of the particle charging in dense plasma crystals are studied very little. For the considered parameters of discharge, numerical simulations [18] predict $\mathcal{A}\sim 0.03$–0.1 for two particles separated horizontally in the sheath by $\Delta\sim\lambda_{\rm De}$. However, direct measurements of the interparticle interaction [6,7] do not show noticeable variation of the particle charge for this separation. Apparently, value of $\mathcal{A}$ does not exceed ~ 0.1 in real experiments, and therefore the squared frequency ω_0^2 of the DL wave [see Eq. (6)] is always positive. It is also difficult to estimate the value of the dispersion $\tilde{\sigma}^2$. The lower limit for the dispersion is determined by the charge discreteness, $\tilde{\sigma}^2\sim Z^{-1}\sim 10^{-3}$–$10^{-4}$ [15,16]. Plasma fluctuations can strongly increase $\tilde{\sigma}^2$, but we do not know reliable measurements of this effect in the sheath. Thus, both $\mathcal{A}$ and $\tilde{\sigma}^2$ are somewhat uncertain parameters.

The damping rate of the DL wave is changed significantly due to delayed charging, when $\mathcal{A}\Omega_{\rm DL}^2/\gamma\Omega_{\rm ch}\gtrsim 1$ [regular variations, see Eq. (6)], or $\tilde{\sigma}^2\Omega_{\rm DL}^2/\gamma\Omega_{\rm ch}\gtrsim 1$ [stochastic variations, see Eq. (16)]. For our conditions, the DL frequency scale varies in the range $\Omega_{\rm DL}\sim 30$–3×10^3 s^{-1} [5], and $\Omega_{\rm DL}^2/\gamma\Omega_{\rm ch}\sim 1$ for $p\sim 10$ Pa and $a\sim 10$ μm. The role the delayed charging increases at low pressures, especially for small particles, since $\gamma\propto p/a$, $\Omega_{\rm DL}^2\propto a^{-1}$, and $\Omega_{\rm ch}\propto pa$ (we suppose $n_e\propto p$), so that $\Omega_{\rm DL}^2/\gamma\Omega_{\rm ch}\propto 1/p^2 a$. Parameter $\mathcal{A}$ presumably does not depend neither on a, nor on p [18]. If we set $\mathcal{A}\sim 10^{-2}$, then the regular variations are expected to increase significantly the damping of the DL wave, when $p\lesssim 3$ Pa and $a\lesssim 1$ μm. The charge dispersion for stochastic fluctuations decreases with a as $\tilde{\sigma}^2\propto a^{-1}$. If $\tilde{\sigma}^2\sim 10^{-3}$ for $a\sim 1$ μm, then the random variations might cause the instability for $p\lesssim 1$ Pa and $a\lesssim 1$ μm.

For vertical oscillations, the role of the delayed charging can be evaluated from Eqs. (12) and (18) for damping rates. We can rewrite these expressions in the following form:

$$\text{Regular:}\quad \mathrm{Im}\,\omega\simeq\gamma-\frac{1}{2}\left(\frac{\ell_E/\ell_Q}{1+\ell_E/\ell_Q}\right)\frac{\Omega_{\rm v}^2}{\Omega_{\rm ch}},$$

$$\text{Random:}\quad \frac{1}{2}\Gamma_{\mathcal{E}}\simeq\gamma-\frac{1}{2}\left(\frac{\tilde{\sigma}}{1+\ell_E/\ell_Q}\right)^2\frac{\Omega_{\rm v}^2}{\Omega_{\rm ch}},$$

where $\ell_Q=Q_0/Q_0'$ and $\ell_E=E_0/E_0'$ are the spatial scales of change of the charge and electric field, respectively. Comparing these expressions we see that the regular (random) variations of charge are more important when the dispersion $\tilde{\sigma}^2$ is much smaller (greater) than the value $\ell_E/\ell_Q+(\ell_E/\ell_Q)^2$. Therefore, if particles are small ($a\lesssim 1$ μm) and are levitated in the presheath, where the charge is practically constant ($\ell_E/\ell_Q\lesssim 10^{-2}$), the stochastic variation of charge may drive an instability of the vertical oscillations, since $\tilde{\sigma}^2$ could be rather high for small particles. If particles are large and trapped below the sheath edge, where the value of ℓ_E/ℓ_Q might be of order of unity, a vertical oscillation instability may be caused by regular charge variations.

For our conditions, the eigenfrequency of vertical oscillations is $\Omega_{\rm v}\sim 10$–10^2 s^{-1}, and $\Omega_{\rm v}^2/\gamma\Omega_{\rm ch}\sim 1$ for $p\sim 3$ Pa and $a\sim 3$ μm. Let us suppose that $E_0'\propto p$ [25]; Then $\Omega_{\rm v}^2\propto p/a^2$, and $\Omega_{\rm v}^2/\gamma\Omega_{\rm ch}\propto 1/pa^2$. If we set $\ell_E/\ell_Q\sim 10^{-1}$ below the sheath edge, then the instability due to regular variations can start for $a\sim 3$ μm at $p\sim 0.3$ Pa. For smaller particle, it is more likely to expect the instability due to stochastic fluctuations: If $a\sim 0.3$ μm, then the pressure threshold is estimated as $p\sim 1$ Pa.

Thus, at low pressures (of order of one Pa) both regular and stochastic variations of charge can influence oscillations of micron size particles in the sheath. Vertical oscillations can be unstable: For small particles, this is due to stochastic fluctuations, whereas for relatively large particles regular variations might be responsible for the instability. Horizontal DL waves are damped by regular variations, but stochastic fluctuations can cause the wave instability. We believe that varying the gas pressure and the particle size, the magnitude of these effects can be measured in experiments.

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